Engineering Mechanics

Index

Equilibrium of Forces & Law of motion and friction

Basic Concept

Mechanics can be defined as the branch of physics concerned with the state of rest or motion of bodies that subjected to the action of forces. OR It may be defined as the study of forces acting on body when it is at rest or in motion is called mechanics.

Mechanics can be divided into two branches.

- Statics It is the branch of mechanics that deals with the study of forces acting on a body in equilibrium. Either the body at rest or in uniform motion is called statics
- Dynamics: It is the branch of mechanics that deals with the study of forces on body in motion is called dynamics. It is further divided into two branches.
  - Kinetics It is the branch of the dynamics which deals the study of body in motion under the influence of force i.e. is the relationship between force and motion are considered or the effect of the force are studied
  - Kinematics: It is the branch of the dynamics that deals with the study of body in motion without considering the force.

Force

- Force In general force is a Push or Pull, which creates motion or tends to create motion, destroy or tends to destroys motion.
- In Engineering mechanics force is the action of one body on another.
- A force tends to move a body in the direction of its action, A force is characterized by its point of application, magnitude, and direction, i.e. a force is a vector quantity.

Units of force

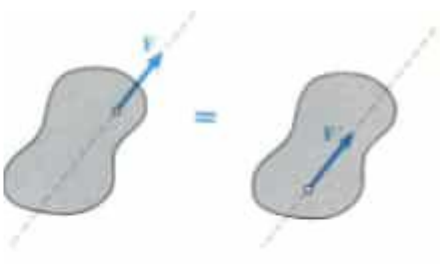
The following force units are frequently used.

- Newton
  - The S.I unit of force is Newton and denoted by N. which may be defined as 1N = 1 kg. 1 m/s2
- Dynes

    - Dyne is the C.G.S unit of force. 1 Dyne = 1 g. 1 cm/s2 One Newton force = 10^5 dyne
- Pounds
    - The FPS unit of force is the pound. 1 lbf = 1 lbm. 1ft/s2 One pound force = 4.448 N One dyne force = 2.248 x 10^{-6} lbs

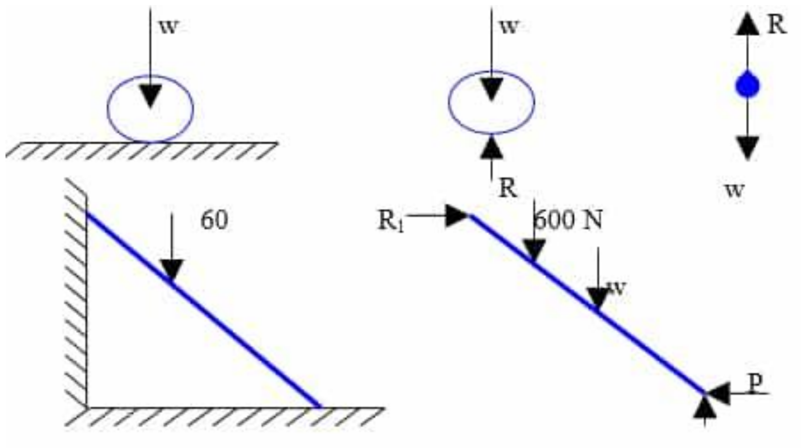
Principle of Transmissibility of forces

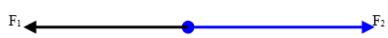
- The state of rest of motion of a rigid body is unaltered if a force acting in the body is replaced by another force of the same magnitude and direction but acting anywhere on the body along the line of action of the replaced force.
- For example the force F acting on a rigid body at point A. According to the principle of transmissibility of forces, this force has the same effect on the body as a force F applied at point B.



Free-Body Diagram:

- A diagram or sketch of the body in which the body under consideration is freed from the contact surface (surrounding) and all the forces acting on it (including reactions at contact surface) are drawn is called free body diagram. Free body diagram for few cases are shown in below





Steps to draw a free-body diagram:

1. Select the body (or part of a body) that you want to analyze, and draw it.
2. Identify all the forces and couples that are applied onto the body and draw them on the body. Place each force and couple at the point that it is applied.
3. Label all the forces and couples with unique labels for use during the solution process.
4. Add any relevant dimensions onto your picture.

Equilibrium: The concept of equilibrium is introduced to describe a body which is stationary or which is moving with a constant velocity. In statics, the concept of equilibrium is usually used in the analysis of a body which is stationary, or is said to be in the state of static equilibrium.

Particles: A particle is a body whose size does not have any effect on the results of mechanical analyses on it and, therefore, its dimensions can be neglected.

Rigid body: A body is formed by a group of particles. The size of a body affects the results of any mechanical analysis on it. A body is said to be rigid when the relative positions of its particles are always fixed and do not change when the body is acted upon by any load (whether a force or a couple).

Force System:

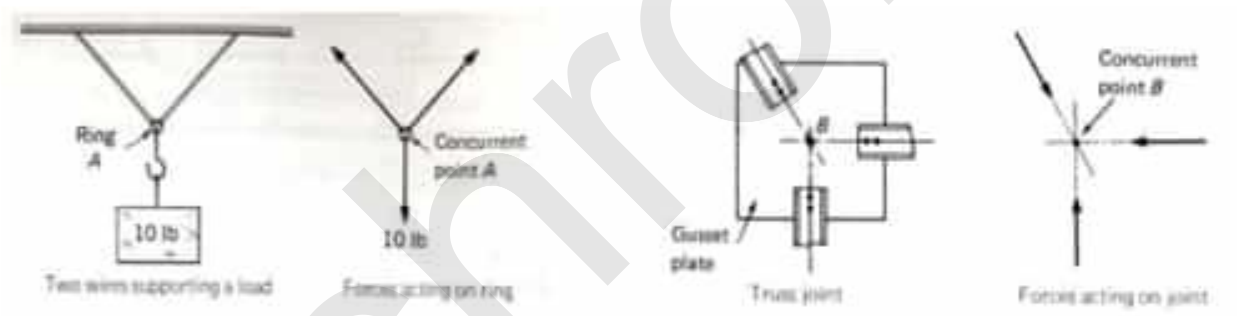
- When a member of forces simultaneously acting on the body, it is known as force system. A force system is a collection of forces acting at specified locations. Thus, the set of forces can be shown on any free body diagram makes-up a force system.

Types of system of forces

- Collinear forces :
  - In this system, line of action of forces act along the same line is called collinear forces. For example consider a rope is being pulled by two players as shown in figure.

- Coplanar forces
  - When all forces acting on the body are in the same plane the forces are coplanar

- Coplanar Concurrent force system
  - A concurrent force system contains forces whose lines of action meet at same one point. Forces may be tensile (pulling) or Forces may be compressive (pushing)



- Non-Concurrent Co-Planar Forces
  - A system of forces acting on the same plane but whose line of action does not pass through the same point is known as non concurrent coplanar forces or system, for example, a ladder resting against a wall and a man is standing on the rung but not on the center of gravity.
- Coplanar parallel forces
  - When the forces acting on the body are in the same plane but their line of actions are parallel to each other known as coplanar parallel forces for example forces acting on the beams and two boys are sitting on the sea saw.
- Non-coplanar parallel forces

- In this case all the forces are parallel to each other but not in the same plane, for example the force acting on the table when a book is kept on it.

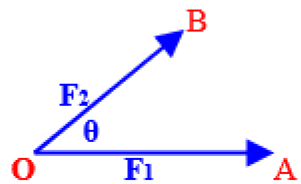
ADDITION OF FORCES

- **ADDITION OF (FORCES) BY HEAD TO TAIL RULE**
    - To add two or more than two vectors (forces), join the head of the first vector with the tail of the second vector, and join the head of the second vector with the tail of the third vector and so on.
    - Then the resultant vector is obtained by joining the tail of the first vector with the head of the last vector. The magnitude and the direction of the resultant vector (Force) are found graphically and analytically.

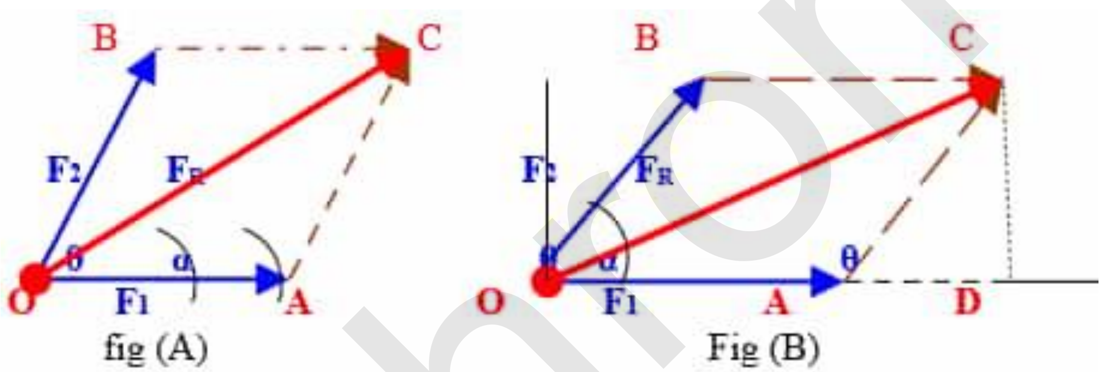
- **RESULTANT FORCE**
    - A resultant force is a single force, which produces same effect so that of number of forces can produce is called resultant force

COMPOSITION OF FORCES

- The process of finding out the resultant Force of given forces (components vector) is called composition of forces. A resultant force may be determined by following methods
    - **PARALLELOGRAM METHOD**
        - According to parallelogram method 'If two forces (vectors) are acting simultaneously on a particle be represented (in magnitude and direction) by two adjacent sides of a parallelogram, their resultant may represent (in magnitude and direction) by the diagonal of the parallelogram passing through the point.
        - The magnitude and the direction of the resultant can be determined either graphically or analytically as explained below.
        - Graphical method Let us suppose that two forces F1 and F2 acting simultaneously on a particle as shown in the figure (a) the force F2 makes an angle θ with force F1



- First of all we will draw a side OA of the parallelogram in magnitude and direction equal to force F1 with some suitable scale. Similarly draw the side OB of parallelogram of same scale equal to force F2, which makes an angle θ with force F1. Now draw sides BC and AC parallel to the sides OA and BC. Connect the point O to Point C which is the diagonal of the parallelogram passes through the same point O and hence it is the resultant of the given two forces. By measurement the length of diagonal gives the magnitude of resultant and angle α gives the direction of the resultant as shown in fig (A)



Analytical method

- In the paralleogram OABC, from point C drop a perpendicular CD to meet OA at D as shown in fig (B)
- In **parallelogram OABC**, **OA = F1 OB = F2** Angle AOB = θ
- Now consider the **ΔCAD** in which **Angle CAD = θ AC = F2**
- By resolving the vector F2 we have, **CD = F2 Sin θ** and **AD = F2 Cosine θ**
- Now consider **ΔOCD** ,Angle **DOC = α**. Angle ODC = 90º

According to Pythagoras theorem ,$(Hyp)^2 = (per)^2 + (base)^2$

- $OC^2 = DC^2 + OD^2$, $OC^2 = DC^2 + (OA + AD)^2$
- $FR^2 = F^2 \sin^2\theta + (F1 + F2 \text{ Cosine } \theta)^2$
- $FR^2 = F^2 2 \sin^2\theta + F^2 1 + F^2 2 \cos^2\theta + 2\, F1\, F2 \text{ Cosine } \theta$.
- $FR^2 = F^2 2 \sin^2\theta + F^2 2 \cos^2\theta + F^2 1 + 2\, F1\, F2 \text{ Cosine } \theta$.
- $FR^2 = F^2 2 (\sin^2\theta + \cos^2\theta) + F^2 1 + 2\, F1\, F2 \text{ Cosine } \theta$.

- $FR^2 = F^2 2\,(1) + F^2 1 + 2\,F1\,F2$ Cosine θ.
- $FR^2 = F^2 2 + F^2 1 + 2\,F1\,F2$ Cosine θ.
- $FR^2 = F^2 1 + F^2 2 + 2\,F1\,F2$ Cosine θ
- $FR = \sqrt{F^2 1 + F^2 2 + 2\,F1\,F2 \text{ Cosine } \theta}$.

TRIANGLE METHOD OR TRIANGLE LAW OF FORCES

- According to triangle law or method" If two forces acting simultaneously on a particle by represented (in magnitude and direction) by the two sides of a triangle taken in order their resultant is represented (in magnitude and direction) by the third side of the triangle taken in opposite order. OR If two forces are acting on a body such that they can be represented by the two adjacent sides of a triangle taken in the same order, then their resultant will be equal to the third side (enclosing side) of that triangle taken in the opposite order. The resultant force (vector) can be obtained graphically and analytically or trigonometry.
- Graphically, Now draw lines OA and AB to some convenient scale in magnitude equal to F1 and F2.
- Join point O to point B the line OB will be the third side of triangle, passes through the same point O and hence it is the resultant of the given two forces.
- By measuring the length of OB gives the magnitude of resultant and angle α gives the direction of the resultant as shown in fig (B).

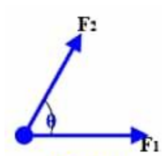


Fig (A)

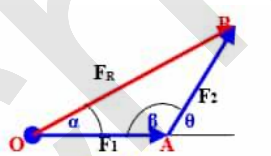
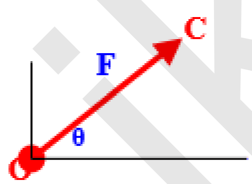
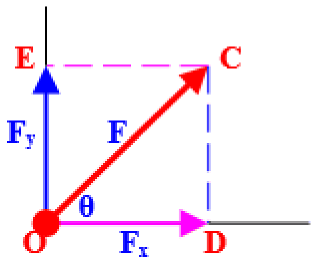


Fig (B)



- Now draw a line OC to represent the vector in magnitude, which makes an angle θ with x-axis with some convenient scale.
- Drop a perpendicular CD at point C which meet x axis at point D, now join point O to point D, the line OD is called horizontal component of resultant vector and represents by Fx in magnitude in same scale.

- Similarly draw perpendicular CE at point C, which will meet y-axis at point E now join O to E. The line OE is called vertical component of resultant vector and represents by Fy in magnitude of same scale



Analytically or trigonometry

In ΔCOD, Angle COD = θ , Angle ODC = 90°

- OC = F
- OD = Fx
- OE = CD = Fy
- We know that Cosine θ = OD/OC. Cosine θ = Fx/F And **Fx = F Cosine θ**
- Similarly we have Sin θ = DC /OC, Sin θ = Fy /F And **Fy = F Sine θ**

Equilibrium Equations for a rigid body:

A rigid acted upon by any applied load will tend to translate dan rotate about a particular axis. The tendency to translate is due to the action of the resultant force on the body and the tendency to rotate is due to the action of the resultant couple.

Equilibrium will occur on the body if the resultant force, as well as the resultant couple, are both zero.

- For equilibrium, the sum of all forces acting on the body is zero.

Resultant Force = ∑F = 0

- The sum of the moment about any axis must be zero.

Resultant Moment = ∑M = 0

Equilibrium Equations in 2D:

The resultant force vector for a planar force system acts on the plane of action of the original force system.

$\sum F = \sum F_x + \sum F_y$

The resultant moment vector acts perpendicular to that plane.

$\sum M_O = \sum |r \times F| = \sum F_d$

where d is the perpendicular distance between any moment centre O and the line of action of F.

The equilibrium equations for the two-dimensional case can be written in the scalar form as follows:

$\sum F_x = 0$

$\sum F_y = 0$

$\sum M_O = 0$

Force System Classification	Equations of Equilibrium		
Coplanar and concurrent	$\Sigma F_x = 0$	$\Sigma F_y = 0$	
Coplanar and non-concurrent	$\Sigma F_x = 0$	$\Sigma M_z = 0$	$\Sigma F_y = 0$
Non-coplanar and concurrent	$\Sigma F_x = 0$	$\Sigma F_y = 0$	$\Sigma F_z = 0$
Non-coplanar and non-concurrent	$\Sigma F_x = 0$	$\Sigma M_x = 0$	$\Sigma F_y = 0$
	$\Sigma M_y = 0$	$\Sigma F_z = 0$	$\Sigma M_z = 0$

Law of Motion

Law 1:

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it. Projectiles continue in their motions, so far as they are not retarded by the resistance of air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are continually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by air. The greater bodies of planets and comets, meeting with less resistance in freer spaces, preserve their motions both progressive and circular for a much longer time.

Law 2:

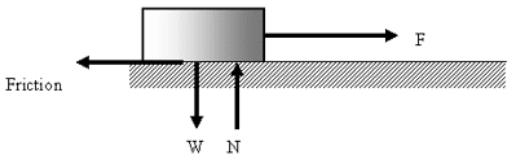
The change of motion is proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed.
If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force is impressed altogether and at once or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.

Law 3:

To every action there is always opposed an equal reaction: or, the mutual action of two bodies upon each other are always equal, and directed to contrary parts. Whatever draws or presses another is as much drawn or pressed by that other. If you press on a stone with your finger, the finger is also pressed by the stone. The Third Law, commonly known as the "action-reaction" law, is the most surprising of the three laws. Newton's great discovery was that when two objects interact, they each exert the same magnitude of force on each other. We shall refer to objects that
interact as an interaction pair.

Friction Force

Frictional forces also exist when there is a thin film of liquid between two surfaces or within a liquid itself. This is known as the viscous force. We will not be talking about such forces and will focus our attention on Coulomb friction i.e., frictional forces between two dry surfaces only. Frictional force always opposes the motion or tendency of an object to move against another object or against a surface. We distinguish between two kinds of frictional forces - static and kinetic - because it is observed that kinetic frictional force is slightly less than maximum static frictional force.



The applied force F, the weight W, the normal reaction of the surface N and the frictional force acting on a block being pulled on a rough surface

Figure 1

The block does not move until the applied force *F* reaches a maximum value Fmax. Thus from F = 0 up to F = Fmax, the frictional force adjusts itself so that it is just sufficient to stop the motion. It was observed by Coulombs that *F max* is proportional to the normal reaction of the surface on the object. You can observe all this while trying to push a table across the room; heavier the table, larger the push required to move it. Thus we can write

$$F_{max} \propto N$$
$$\text{or} \quad F_{max} = \mu_s N$$

where μs is known as the coefficient of static friction. It should be emphasize again that is the maximum possible value of frictional force, applicable when the object is about to stop, otherwise frictional force could be less than, just sufficient to prevent motion. We also note that frictional force is independent of the area of contact and depends only on *N* .

As the applied force *F* goes beyond *F* , the body starts moving now experience slightly less force compound to. This force is seem to be when is known as the coefficient of kinetic friction. At low velocities it is a constant but decrease slightly at high velocities. A schematic plot of frictional force *F* as a function of the applied force is as shown in figure 2.

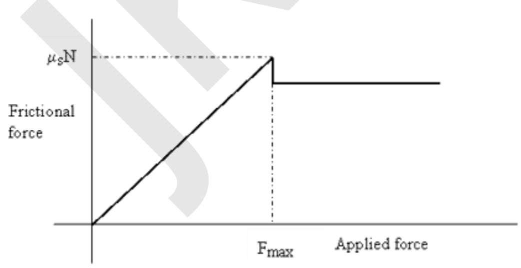
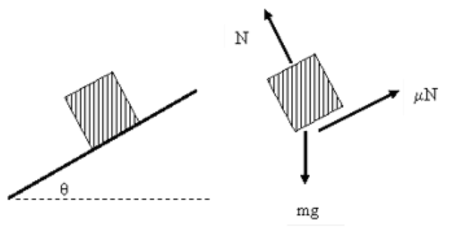


Figure 2

Values of frictional coefficients for different materials vary from almost zero (ice on ice) to as large as 0.9 (rubber tire on cemented road) always remaining less than 1.

A quick way of estimating the value of static friction is to look at the motion an object on an inclined plane. Its free-body diagram is given in figure 3.



A block of mass m on an inclined plane (left) and its free-body diagram (right) when it is about to slide down the ramp

Figure 3

Since the block has a tendency to slide down, the frictional force points up the inclined plane. As long as the block is in equilibrium

$$mg \sin\theta \leq maximum\ friction$$
$$mg\cos\theta = N$$

As θ is increased, mgsinθ increases and when it goes past the maximum possible value of friction fmax the block starts sliding down. Thus at the angle at which it slides down we have,

$$\left.\begin{aligned} mg\sin\theta = f_{max} &= \mu_s N \\ &= \mu_s mg\cos\theta \end{aligned}\right\} \Rightarrow \mu_s = \tan\theta$$

Impulse and Momentum

Linear Momentum and its Conservation

The linear momentum of a particle of mass m moving with velocity ¯v is defined as

$$\vec{p} = m\vec{v}$$

- Linear momentum, being the product of a scalar and a vector, is a vector.
- It has dimensions M.L.T–1 and units $kg.m.s^{-1}$.
- The direction of the momentum vector is the same as the velocity vector.

We can express Newton's second law in terms of linear momentum in this way

$$\sum \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$$

$$= m\frac{d\vec{v}}{dt} = m\vec{a}.$$

- Thus the resultant force on an object (system) equals the time rate of change of linear momentum of the object (system).
- The definition of linear momentum enables us to put the second law into a more general powerful form.
- If, in addition, the system is an isolated one then we can formulate a law of conservation of linear momentum.

Impulse and Momentum

- Impulse is defined as simply change in momentum. In a collision between two particles (and especially a contact collision) the force of interaction might vary with time.
- The force is relatively short-lived, being zero before clock time ti and zero after clock time tf and having a relatively large value at maximum.
- The elapsed time for the interaction is to a good approximation Δt = tf – ti.

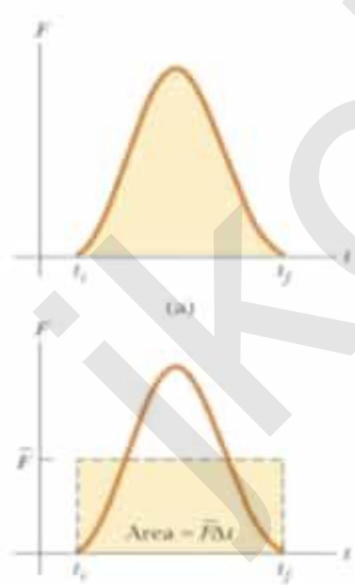


fig above shows A force that varies over a relatively short elapsed time. The area under the force curve is equal to the magnitude of the impulse

- The sum of forces is a function of time.
- To find the change in momentum we must integrate over the elapsed time for the interaction:

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \sum \vec{F}(t)dt.$$

- The change in momentum is defined as the impulse and given the symbol ¯J

$$\vec{J} = \int_{t_i}^{t_f} \sum \vec{F}(t)dt = \Delta\vec{p}.$$

- Impulse has the same dimensions and units as momentum and is also a vector.
- The impulse has a magnitude equal to the area under the force curve between the two clock times, that is, over the elapsed time of the collision.
- The direction of the impulse vector is the same as the direction of the change in momentum vector.

By Newton's Second Law, the net force is equal to the mass of object times its acceleration,

$$\sum \vec{F} = m\vec{a}$$

This can be substituted into the equation for impulse,

$$\vec{J} = \sum \vec{F}\,\Delta t$$

$$\therefore \vec{J} = m\left(\frac{\Delta\vec{v}}{\Delta t}\right)\Delta t$$

$$\therefore \vec{J} = m\left(\frac{\Delta\vec{v}}{\Delta t}\right)\Delta t$$

$$\therefore \vec{J} = m\Delta\vec{v}$$

The change in velocity is the difference between the velocities at the starting and ending times,

$$\Delta\vec{v} = \overrightarrow{v_2} - \overrightarrow{v_1}$$

The formula for impulse becomes,

$$\vec{J} = m(\vec{v_2} - \vec{v_1})$$

$$\therefore \vec{J} = m\vec{v_2} - m\vec{v_1}$$

$$\therefore \vec{J} = \vec{p_2} - \vec{p_1}$$

- This equation is called the ***impulse-momentum theorem***. In words, it states that the change in momentum of an object in a certain time interval is equal to the impulse of the net force that acts on the object in the time interval.
- Using this formula, it is possible to relate changes in momentum to the forces that were applied to cause the change.
- It also shows that the time over which a force is applied has an effect on the change in momentum that results.
- It is also important to note that the units for momentum and impulse are effectively the same. The unit of momentum is kg m/s, and the unit of impulse is the Newton-second, N.s.

Conservation of Momentum

- When two objects interact, such as in a collision, they may exert forces on each other.
- The forces the objects exert on each other can be considered part of a *closed* or *isolated* system. In this case, the forces involved are *internal forces*.
- If any outside forces affect the system, these are called *external forces*.
- According to Newton's Third Law, when there are no external forces, the internal forces that act between two objects have equal magnitudes and opposite directions. If the two objects are labelled *A* and *B*, the forces they exert on each other are,

$$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$$

- During a collision, these forces act for the same amount of time. If the collision begins at time t1 and ends at time t2, then the time duration of the collision is Δt , and the impulse experienced by object A is,

$$\vec{J_A} = \vec{F}_{B\ on\ A}\Delta t$$

$$\vec{J_B} = \vec{F}_{A\ on\ B}\Delta t$$

- The impulse experienced by object B is J_B,

- If the values for the forces in these impulse equations are substituted in to the equation for Newton's Third Law, the result is,

$$\vec{F}_{A\ on\ B} = -\vec{F}_{B\ on\ A}$$

$$\therefore \frac{\vec{J_B}}{\Delta t} = -\frac{\vec{J_A}}{\Delta t}$$

$$\therefore \vec{J_B} = -\vec{J_A}$$

By the impulse-momentum theorem, this is equivalent to,

$$\overrightarrow{p_{B,2}} - \overrightarrow{p_{B,1}} = -(\overrightarrow{p_{A,2}} - \overrightarrow{p_{A,1}})$$

- In this equation, $P_{A,1}$ means the momentum of object A at time t_1 , $P_{A,2}$ means the momentum of object A at time t_2 , $P_{B,1}$ means the momentum of object B at time t1, and $P_{B,2}$ means the momentum of object B at time t2.
- The equation can be rearranged to put all of the terms for time t1 on one side, and terms for time t2 on the other,

$$\overrightarrow{p_{A,1}} + \overrightarrow{p_{B,1}} = \overrightarrow{p_{A,2}} + \overrightarrow{p_{B,2}}$$

- In this case, in which there were no external forces, the sum of the momenta before the collision is equal to the sum of the momenta after.
- In general, as long as there are no external forces, the total momentum of the system is constant. This is known as *conservation of momentum*.
- For any number of objects the total momentum can be labeled $P_{A,2}$,

$$\vec{P} = \overrightarrow{p_A} + \overrightarrow{p_B} + \cdots = m_A\overrightarrow{v_A} + m_B\overrightarrow{v_B} + \cdots$$

If there are no external forces, the total momentum P remains constant, even if the moment of the individual objects change.

Instantaneous Impulse

Example: bat and ball contact

$$J = \int F.dt \Rightarrow \triangle \rho = \rho_f - \rho_i$$

- The relation between impulse and linear momentum can be understood by the following equation.

$$Ft = m(v - u)$$

Where, *F* = Force, *t* = time, *m* = mass, *v* = initial velocity, *u* = final velocity

- Rotation about a fixed point gives the three-dimensional motion of a rigid body attached at a fixed point.

Theory of Collision

- A **collision** occurs when two or more objects hit each other. When objects collide, each object feels a force for a short amount of time.
- This force imparts an impulse or changes the momentum of each of the colliding objects.
- But if the system of particles is isolated, we know that momentum is conserved.
- Therefore, while the momentum of each individual particle involved in the collision changes, the total momentum of the system remains constant.

The collision between two bodies may be classified in two ways: Head-on collision, and Oblique collision.

- **Head-on Collision**
    - Let the two balls of masses *m*1 and *m*2 collide directly with each other with velocities *v*1 and *v*2 in the direction as shown in figure.
    - After the collision, the velocity becomes and along the same line.

- **Oblique Collision**
    - In case of oblique collision linear momentum of an individual particle do change along the common normal direction.
    - No component of impulse act along the common tangent direction.
    - So, linear momentum or linear velocity remains unchanged along tangential direction. Net momentum of both the particle remains conserved before and after collision in any direction.

General Equation for Velocity after Collision

$$v_1' = \left(\frac{m_1 - em_2}{m_1 - m_2}\right)v_1 + \left(\frac{m_2 + em_2}{m_1 + m_2}\right)v_2$$

$$v_2' = \left(\frac{m_2 - em_2}{m_1 + m_2}\right)v_2 + \left(\frac{m_1 + em_2}{m_1 + m_2}\right)v_1$$

Where m1 = mass of body 1

- m2 = mass of body 2
- v1 = velocity of body 1
- v2 = velocity of body 2= velocity of body 1 after collision= velocity of body 2 after collision

Where e = coefficient restitution

- In case of head-on elastic collision *e* = 1
- In case of head-on inelastic collision 0 < *e* < 1
- In case of head-on perfectly inelastic collision *e* = 0

The procedure for analyzing a collision depends on whether the process is **elastic** or **inelastic**. Kinetic energy is conserved in elastic collisions, whereas kinetic energy is converted into other forms of energy during an inelastic collision. In both types of collisions, momentum is conserved.

- **Elastic Collisions**
    - Some kinetic energy is converted into sound energy when pool balls collide otherwise, the collision would be silent and a very small amount of kinetic energy is lost to friction.
    - However, the dissipated energy is such a small fraction of the ball's kinetic energy that we can treat the collision as elastic.

Equations for Kinetic Energy and Linear Momentum

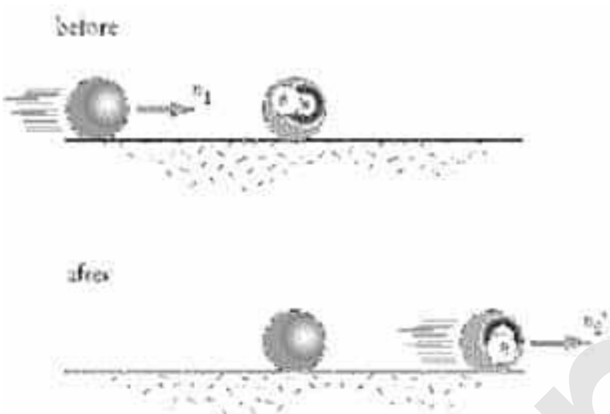
- Let's examine an elastic collision between two particles of mass m_1 and m_2, respectively. Assume that the collision is head-on, so we are dealing with only one dimension—you are unlikely to find two-dimensional collisions of any complexity on SAT II Physics. The velocities of the particles before the elastic collision are v_1 and v_2, respectively. The velocities of the particles after the elastic collision are v_1' and v_2'. Applying the law of conservation of kinetic energy, we find:

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$$

Applying the law of conservation of linear momentum:

$$m_1 v_1 + m_2 v_2 = m_1 v_1\prime + m_2 v_2\prime$$

- These two equations put together will help you solve any problem involving elastic collisions. Usually, you will be given quantities for m_1, m_2, v_1 and v_2, and can then manipulate the two equations to solve for v_1' and v_2'
- A head-on with the cue ball in pool, Both of these balls have the same mass, and the velocity of the cue ball is initially v_1. What are the velocities of the two balls after the collision? Assume the collision is perfectly elastic



Substituting $m_1 = m_2 = m$ and $v_2 = 0$ into the equation for conservation of kinetic energy we find:

$$\frac{1}{2} m v_1^2 = \frac{1}{2} m (v_1\prime^2 + v_2\prime^2)$$
$$v_1^2 = v_1\prime^2 + v_2\prime^2$$

Applying the same substitutions to the equation for conservation of momentum, we find:

$$m v_1 = m v_1\prime + m v_2\prime$$
$$v_1 = v_1\prime + v_2\prime$$

If we square this second equation, we get:

$$v_1^2 = v_1\prime^2 + v_2\prime^2 + 2 v_1\prime v_2\prime$$

By subtracting the equation for kinetic energy from this equation, we get:

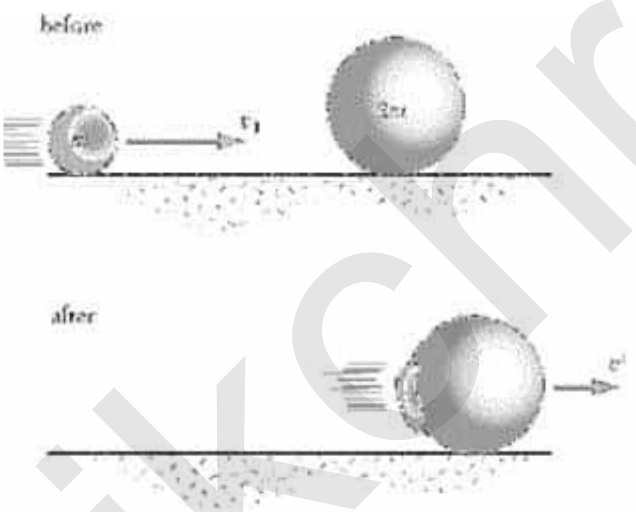
$$2 v_1\prime v_2\prime = 0$$

- The only way to account for this result is to conclude that $v_1' = 0$ and consequently. $v_1 = v_2'$

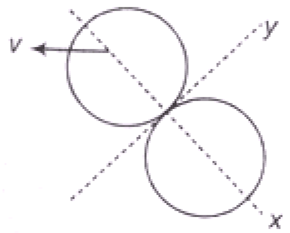
Inelastic Collisions

- Most collisions are inelastic because kinetic energy is transferred to other forms of energy—such as thermal energy, potential energy, and sound—during the collision process.
- The kinetic energy is not conserved, in inelastic collision. Momentum is conserved in all inelastic collisions.
- The one exception to this rule is in the case of **completely inelastic collisions**.
- **Completely Inelastic Collisions**
    - A completely inelastic collision also called a "perfectly" or "totally" inelastic collision, is one in which the colliding objects stick together upon impact.
    - As a result, the velocity of the two colliding objects is the same after they collide.
    - Because $v_1' = v_2' = v'$, question may be asked for finding

for eg. below two gumballs, of mass *m* and mass 2*m* respectively, collide head-on. Before impact, the gumball of mass *m* is moving with a velocity v_1, and the gumball of mass 2*m* is stationary.



- First, note that the gumball wad has a mass of *m* + 2*m* = 3*m*. The law of conservation of momentum tells us that $mv_1 = 3mv'$, and so $v' = v_1/3$. Therefore, the final gumball wad moves in the same direction as the first gumball, but with one-third of its velocity.



Oblique collision

Kinematics and Dynamics of Particles and Rigid Bodies

Plane Motion: When all parts of the body move in a parallel plane then a rigid body said to perform plane motion.

- The motion of rigid body is said to be translation, if every line in the body remains parallel to its original position at all times.
- In translation motion, all the particles forming a rigid body move along parallel paths.
- If all particles forming a rigid body move along parallel straight line, it is known as rectilinear translation.
- If all particles forming a rigid body does not move along a parallel straight line but they move along a curve path, then it is known as curvilinear translation.

Straight Line Motion: It defines the three equations with the relationship between velocity, acceleration, time and distance travelled by the body. In straight line motion, acceleration is constant.

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

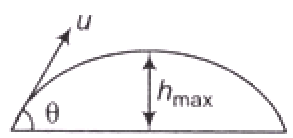
Where, *u* = initial velocity, *v* = final velocity, *a* = acceleration of body, *t* = time, and *s* = distance travelled by body.

Distance travelled in *n*th second:

$$s_n = u + \frac{1}{2}a(2n - 1)$$

Projectile Motion: Projectile motion defines that motion in which velocity has two components, one in horizontal direction and other one in vertical direction. Horizontal component of velocity is constant during the flight of the body as no acceleration in horizontal direction.

Let the block of mass is projected at angle θ from horizontal direction.



Projectile motion

Maximum height

$$h_{max} = \frac{u^2 \sin^2 \theta}{2g}$$

Time of flight

$$T = \frac{2u \sin \theta}{g}$$

Range

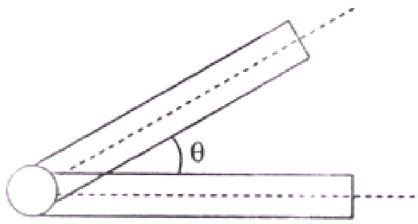
$$R = \frac{u^2 \sin 2\theta}{g}$$

Where, *u* = initial velocity.

- At maximum height vertical component of velocity becomes zero.
- When a rigid body move in circular path centred on the same fixed axis, then the particle located on axis of rotation have zero velocity and zero acceleration.
- Projectile motion describes the motion of a body, when the air resistance is negligible.

Rotational Motion with Uniform Acceleration: Uniform acceleration occurs when the speed of an object changes at a constant rate. The acceleration is the same over time. So, the rotation motion with uniform acceleration can be defined as the motion of a body with the same acceleration over time.

Let the rod of block rotation about a point in horizontal plane with angular velocity.



Rotational motion

Angular

$$\omega = \frac{d\theta}{dt}$$

velocity (change in angular displacement per unit time)

Angular acceleration

$$\alpha = \frac{d\omega}{dt} \quad \Rightarrow \quad \alpha = \frac{d^2\theta}{dt^2}$$

Where θ = angle between displacement.

In case of angular velocity, the various equations with the relationships between velocity, displacement and acceleration are as follows.

$$\theta = \omega t$$

$$\alpha = 0$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

Where ω_0 = initial angular velocity, ω = final angular velocity, α = angular acceleration, and θ = angular displacement.

Angular displacement in *n*th second:

$$\theta_n = \omega_0 + \frac{1}{2}\alpha(2n-1)$$

Relation between Linear and Angular Quantities

There are following relations between linear and angular quantities in rotational motion.

$$|e_r| = |e_t| = 1$$

e_r and e_t are radial and tangential unit vector.

Linear velocity

$$v = r\omega\ e_t$$

Linear acceleration (Net)

$$a = \omega^2 r\, e_r + \frac{dv}{dt} e_t$$

Tangential acceleration

$$a_t = \frac{dv}{dt}$$

(rate of change of speed)

Centripetal acceleration

$$a_r = \omega^2 r = \frac{v^2}{r} \quad (\because v = r\omega)$$

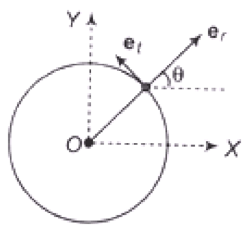
Net acceleration,

$$a = \sqrt{a_r^2 + a_t^2}$$

$$= \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

Where a_r = centripetal acceleration

a_t = tangential acceleration



Position of radial and tangential vectors

Centre of Mass of Continuous Body: Centre of mass of continuous body can be defined as

- Centre of mass about

$$x, x_{CM} = \frac{\int x dm}{\int dm} = \frac{\int x\ dm}{M}$$

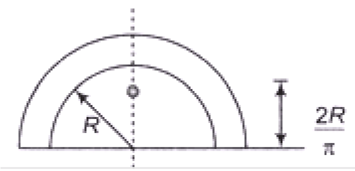
- Centre of mass about

$$y, y_{CM} = \frac{\int y dm}{\int dm} = \frac{\int y\ dm}{M}$$

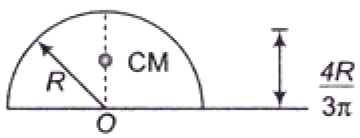
- Centre of mass about

$$z, z_{CM} = \frac{\int z\, dm}{\int dm} = \frac{\int z\, dm}{M}$$

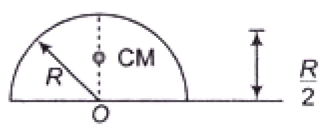
- CM of uniform rectangular, square or circular plate lies at its centre.
- CM of semicircular ring



- CM of semicircular disc



- CM of hemispherical shel



- CM of solid hemisphere

Law of Conservation of Linear Momentum

The product of mass and velocity of a particle is defined as its linear momentum (p).

$$p = mv$$
$$p = \sqrt{2Km}$$
$$F = \frac{dp}{dt}$$

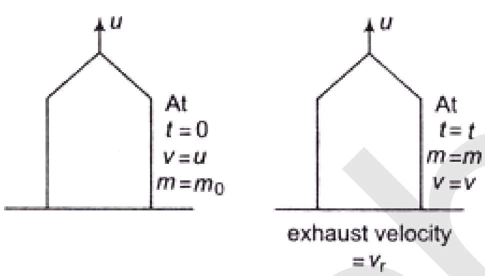
Where, *K* = kinetic energy of the particle

F = net external force applied to body

P = momentum

Rocket Propulsion

Let m_0 be the mass of the rocket at time $t = 0$, *m* its mass at any time *t* and v its velocity at that moment. Initially, let us suppose that the velocity of the rocket is *u*.



Rocket propulsion

- Thrust force on the rocket

$$F_t = v_r\left(-\frac{dm}{dt}\right)$$

Where,

$$-\frac{dm}{dt} =$$

rate at which mass is ejecting

v_r = relative velocity of ejecting mass (exhaust velocity)

- Weight of the rocket w = mg
- Net force on the rocket
- $F_{net} = F_t - w = v_r\left(\frac{-dm}{dt}\right) - mg$
- Net acceleration of the rocket

$$a = \frac{F}{m}$$

$$\frac{dv}{dt} = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$$

$$v = u - gt + v_r \, In \, \frac{m_0}{m}$$

Where, m_0 = mass of rocket at time $t = 0$

m = mass of rocket at time.